ABSTRACTS

List of Talks

Pierre Calka: Convex hulls of perturbed random point sets

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Based on the joint work with J. E. Yukich (Lehigh University, United States)
We consider the convex hull of a perturbed point process comprised of i.i.d. points distributed as the sum of a uniform point on the unit sphere and a uniform point in the ball centered at the origin and of radius n^α. This model, inspired by the smoothed complexity analysis introduced in computational geometry is a perturbation of the classical random polytope model. We investigate the scaling limit of the point process and deduce from it explicit expectation and variance asymptotics, as well as central limit theorems for the number of k-dimensional faces of the convex hull. The growth rates depend on the exponent α and we identify the exact phase transitions.It time permits, we will also discuss the situation when the uniform perturbation is replaced by a Gaussian perturbation.

References

[1] P. Calka and J. E. Yukich. Convex hulls of perturbed random point sets. *in preparation*, 2019.

[2] O. Devillers, M. Glisse, X. Goaoc and R. Thomasse. Smoothed complexity of convex hulls by witnesses and collectors. *J. of Comput. Geometry*, 7:101–144, 2016.

Nicolas Chenavier: The maximal degree in a Poisson-Delaunay graph

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Based on the joint work with Gilles Bonnet (Ruhr-Universität Bochum)
A Delaunay triangulation associated with a locally finite subset χ in $\mathbf{R}^2$ is a triangulation $DT(\chi)$ such that no point in χ belongs to the interior of the circumdisk of any triangle in $DT(\chi)$. This model is the key ingredient of the first algorithm for computing the minimal spanning tree and is extensively used in various domains, such as medical image segmentation and finite element methods to build meshes.
In this talk, we consider the case where $\chi = \eta$ is a homogeneous Poisson point process in $\mathbf{R}^2$. We investigate the maximal degree Δ_n of the so-called Delaunay graph associated with η (consisting of the set of edges of triangles) observed in the window $W_n = [-n, n]^2$, namely

$$\Delta_n = \max_{x \in \eta \cap W_n} d_\eta(x),$$

where $d_\eta(x)$ is the degree of any point x, i.e. the number of edges passing through x. As n goes to infinity, we show that Δ_n is concentrated on two consecutive integers. We also provide the exact order of Δ_n.

Simon Coste: Spectra of unimodular trees

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Based on the joint work with Justin Salez (Universit Paris-Dauphine)
The empirical spectral measure of the adjacency matrix in several models of random graphs (such as the Erds-Rnyi model with fixed mean degree) converges towards a limiting distribution, which can often be represented as the spectral measure at the root of an infinite tree exhibiting a special property, unimodularity. In this general talk, I will present this convergence, its link with unimodularity, and a list of questions on the spectra of several infinite unimodular trees along with some answers.

References

[1] Charles Bordenave, Arnab Sen and Balint Virag Mean quantum percolation JEMS, 2017

[2] Simon Coste and Justin Salez Emergence of extended states at zero in the spectrum of sparse random graphs arxiv.org/abs/1809.07587, 2018

[3] Justin Salez Spectral atoms of unimodular random trees JEMS, 2019

David Coupier: Absence of percolation for Poisson outdegree-one graphs

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Based on joint works with David Dereudre (Univ. Lille), Jean-Baptiste Gouéré (Univ. Tours) and Simon Le Stum (Univ. Lille)
Let us describe the Line Segment model. From any point of a (homogeneous and planar) Poisson process, a unilateral segment is growing according to a random direction chosen uniformly on the sphere. The growth speeds of segments are equal and their directions are picked independently. Each segment stops its growth whenever it hits another segment while the stopping segment continues to grow. Hence, the collection of (stopped) segments create connected components. In [**?**], D. J. Daley et al proved that the Line Segment model is well defined and they conjectured that each connected component is bounded (absence of percolation).
The Line Segment model can be actually interpreted as a Poisson outdegree-one graph (POG), i.e. an oriented graph based on a Poisson point process such that each vertex has only one outgoing edge. The Lylipond model is another example of POG, known for having only bounded connected components. In this talk, we state that any POG satisfying two explicit assumptions, called the Loop and Shield assumptions, does not percolate. Thus we obtain the conjecture of [**?**] by proving that the Line Segment model fulfills the Loop and Shield assumptions.
Finally, these two assumptions are discussed and some extensions are investigated.

References

[1] D. J. Daley, S. Ebert and G. Last. Two lilypond systems of finite line-segments. *Probab. Math. Statist.*, 36(2):221–246, 2016.

Serguei Dachian: On the relationship of energy and probability in statistical physics

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Based on the joint work with Boris Nahapetian (Institute of Mathematics, National Academy of Sciences of the Republic of Armenia)

We present a new point of view on the mathematical foundations of statistical physics of infinite volume systems. This viewpoint is based on the newly introduced notions of transition energy field and of one-point transition energy field, which are defined axiomatically by their natural and physically well-founded intrinsic properties. The developed approach allows to give a proper mathematical definition of the Hamiltonian without involving the notion of potential, to propose a justification of the Gibbs formula for infinite systems, to give a simple proof of the solution proposed earlier by the authors to Dobrushin's problem of description of specifications by means of systems of one-point probability distributions, as well as to answer the problem stated by D. Ruelle of how wide the class of specifications, which can be represented in Gibbsian form, is.

Daniela Flimmel: Asymptotic properties of cylinder processes driven by stationary Brillinger-mixing point processes

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Based on the joint work with Lothar Heinrich (Augsburg University)

Let us have a stationary independently marked Brillinger-mixing point process on the real line with two mutually independent marks. Each marked point is interpreted as a distance from the origin assigned with an orientation and thickness of the planar cylinder described by this triplet. We are interested in the asymtotic behaviour of the union set of those parts of cylinders that are observed in some convex set increasing to the full plane. Some asymptotic result are shown for the general case of a stationary Brillinger-mixing underlying point process (e.g. Poisson cluster process with stationary parent point process or stationary determinantal point process). Some mild assumptions on the distributions of the orientation and cylinder thickness are also needed.

Sergey Foss: Interference Queueing Networks on Grids

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Based on the joint work with Abishek Sankararaman (UT Austin) and Francois Baccelli (UT Austin and INRIA

Consider a countably infinite collection of interacting queues, with a queue located at each point of the d-dimensional integer grid, having independent Poisson arrivals, but dependent service rates. The service discipline is of the processor sharing type, with the service rate in each queue slowed down, when the neighbouring queues have a larger workload. The interactions are translation invariant in space and is neither of the Jackson Networks type, nor of the mean-field type. Coupling and percolation techniques are first used to show that this dynamics has well defined trajectories. Coupling from the past techniques are then proposed to build its minimal stationary regime. The rate conservation principle of Palm calculus is then used to identify the stability condition of this system, where the notion of stability is appropriately defined for an infinite dimensional process. We show that the identified condition is also necessary in certain special cases and conjecture it to be true in all cases. Remarkably, the rate conservation principle also provides a closed form expression for the mean queue size. When the stability condition holds, this minimal solution is the unique translation invariant stationary regime. In addition, there exists a range of small initial conditions for which the dynamics is attracted to the minimal regime. Nevertheless, there exists another range of larger though finite initial conditions for which the dynamics diverges, even though stability criterion holds.

References

[1] Abishek Sankararaman, Francois Baccelli, Sergey Foss. Interference Queueing Networks on Grids. *Ann. Appl. Probab.*, 2019 (to appear).

Francesca Greselin: Inferential results for a new inequality curve

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Based on the joint work with Youri Davydov (St. Petersburg State University)

A new integrated inequality curve, essentially designed for capturing significant shifts in the left and right tails of income distributions, has been introduced recently in the literature. The curve arises when we are considering, say, the $p\%$ of the less fortunate people, and we want to measure how their average income differs from the average income of the $p\%$ of the wealthiest, more fortunate persons. The greater the difference, the higher the inequality. In this talk we present inferential results for this integrated curve. We introduce two estimators for it and show that they are asymptotically equivalent and consistent. The main result of the work consists in the proof of their weak convergence to some limiting Gaussian process. An application to real data is also presented.

References

[1] Y. Davydov and F. Greselin *Comparisons between poorest and richest to measure inequality.* Sociological Methods & Research, OnlineFirst(0), 0049124117747300, 2018.

[2] Y. Davydov and F. Greselin *Inferential results for a new measure of inequality.* The Econometrics Journal, 22 (2), 153172, https://doi.org/10.1093/ectj/utz004, 2019.

[3] F. Greselin, L. Pasquazzi and R. Zitikis *Zenga's new index of economic inequality, its estimation, and an analysis of incomes in Italy.* Journal of Probability and Statistics, 26 pp., DOI:10.1155/2010/718905

Anna Gusakova: Concentration inequalities for functionals of Poisson cylinder processes

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Based on the joint work with Anastas Baci (Ruhr University Bochum, Germany), Carina Betken (Ruhr University Bochum, Germany) and Christoph Thäle (Ruhr University Bochum, Germany)

In this talk we consider a random union sets Z associated with stationary Poisson processes of k-cylinders in $\mathbb{R}^d$. Under general conditions on the typical cylinder base we derive the concentration inequalities for the volume of Z restricted to a compact window. Assuming convexity of the typical cylinder base and isotropy of Z we establish a concentration inequality for intrinsic volumes of order $j > k$. Moreover we discuss a number of special cases, for example the case when the cylinder bases arise from a random rotation of a fixed convex body, and the situation of expanding windows. Special attention is payed to the case $k = 0$, which corresponds to the classical Boolean model.

References

[1] A. Baci, C. Betken, A Gusakova and C. Thäle.

Concentration inequalities for functionals of Poisson cylinder processes.

arXiv:1908.021121908.02112.

Raphaël Lachièze-Rey: Percolation of shot-noise excursions

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Based on the joint work with Stephen Muirhead (Queen Mary University of London)
Stationary shot noise fields are a special class of infinitely divisible random fields in the Euclidean space, they can be seen as a spatial moving average based on a homogeneous Poisson measure. We consider the excursion sets of a planar symmetric shot noise field $\{f(x), x \in \mathbb{R}^2\}$:

$$E_u(f) = \{x \in \mathbb{R}^2 : \ f(x) \geqslant u\},$$

for $u \in \mathbb{R}$. In particular, we consider wether $E_u(f)$ percolates, under assumptions involving the smoothness of f and the decay of its correlation function at infinity. We find results which are similar to the Gaussian case [1], under analogous hypotheses: $E_u(f)$ percolates only at $u < 0$, and there is a sharp phase transition at 0.

References

[1] S. Muirhead and H. Vanneuville *The sharp phase transition for level set percolation of smooth planar Gaussian fields*. to appear in Ann IHP, 2018+.

Günter Last: Percolation properties of the random connection model

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In the first part of the talk we will use the Stein-Malliavin method to derive optimal Berry-Esseen bounds for the normal approximation of the number of clusters in a given observation window. In the second part of the talk we first introduce and discuss the lace expansion of the two-point function of the model. Under further assumptions we can then use techniques from point process and continuum percolation theory to establish mean-field behavior above a critical dimension or for certain-long models. In particular there is no percolation at the critical intensity.
This talk is based on the two papers listed below.

References

[1] G. Last, F. Nestmann, M. Schulte. The random connection model and functions of edge-marked Poisson processes: second order properties and normal approximation. arXiv:1808.01203, 2018.

[2] M. Heydenreich, R. van der Hofstad, G. Last, K. Matzke. Lace expansion and mean-field behavior for the random connection model. arXiv:1908.11356, 2019.

Alexander Litvak: Singularity of random Bernoulli 0/1 matrices

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Based on the joint work with Konstantin Tikhomirov (The Georgia Institute of Technology, USA)
Let M be a random $n \times n$ matrix, whose entries are independent $0/1$ random variables taking value 1 with probability $p = p_n$. We provide sharp bounds on the probability that M is singular. Roughly speaking, this probability is comparable with the probability that M has either zero row or zero column.

Julien Randon-Furling: Convex hulls of several Gaussian random walks

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Based on the joint work with Dmitry Zaporozhets (St. Petersburg Department of Steklov Mathematical Institute)
We derive an explicit formula for the expected volume and the expected number of faces of the convex hull of several multidimensional Gaussian random walks.

Filip Seitl: Exploration of Gibbs-Laguerre tessellations for 3D stochastic modeling

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Based on the joint work with Lukas Petrich (Ulm University, Institute of Stochastics), Carl E. Krill III (Ulm University, Institute of Functional Nanosystems), Volker Schmidt (Ulm University, Institute of Stochastics), Daniel Jahn (Charles University, Faculty of Mathematics and Physics) and Viktor Beneš (Charles University, Faculty of Mathematics and Physics)
Random tessellations in Euclidean spaces are models used in various areas of physics, biology, etc. Frequently the tessellation is generated by a locally finite configuration of points called generators. The simplest model, called Voronoi tessellation, splits the space to polyhedra, called cells, in order to associate all locations in that space with the closest generator with respect to the Euclidean metric. The randomnes is inherited by assuming that the set of generators is formed by a point process. Gibbs point processes are good candidates since they enable us to incorporate interactions between points. These interactions are driven by the energy function which together with the activity parameter determines the Gibbs model. The dependence of the tessellation geometry on the energy function allows to create more complex tessellations. The Gibbs-Voronoi tessellations in 2D space were examined in [2]. We deal with an extension of this concept, namely the Gibbs-Laguerre tessellation in 3D. This is motivated by materials research where the 3D grain microstructure of polycrystalline metals is of the interest. The Laguerre tessellations [3] are more suitable than

Voronoi tessellations since they offer more variability in polyhedral cell shapes and in their spatial distribution.
Conditions for the existence of the infinite volume Gibbs-Laguerre tessellation are derived using a general method from [1]. Since these models are hardly analytically tractable, their simulations are of the interest, which are performed by Markov chain Monte Carlo (MCMC) techniques. During the run of algorithm the tessellation geometry is recomputed locally. This together with increasing computer performance make the simulations tractable. Finally, special forms of the energy function lead to statistical reconstruction, inspired by [5]. This method is based on a real data specimen of a polycrystal. The aim is to obtain similar realizations with prescribed geometrical properties. This extends the procedure of finding a deterministic Laguerre tessellation to the experimental data as described in [4].

References

[1] D. Dereudre, R. Drouilhet and H. O. Georgii. Existence of Gibbsian point processes with geometry-dependent interactions. *Probab. Theory Relat. Fields*, 153(3-4):643–670, 2012.

[2] D. Dereudre and F. Lavancier. Practical simulation and estimation for Gibbs Delaunay-Voronoi tessellations with geometric hardcore interaction. *Comput. Stat. Data Anal.*, 55(1):498–519, 2011.

[3] C. Lautensack and S. Zuyev. Random Laguerre tessellations. *Adv. Appl. Probab.*, 40(3):630–650, 2008.

[4] A. Spettl, T. Brereton, Q. Duan, T. Werz, C. E. Krill III, D. Kroese and V. Schmidt. Fitting Laguerre tessellation approximations to tomographic image data. *Philos. Mag.*, 96(2):166–189, 2016.

[5] A. Tscheschel and D. Stoyan. Statistical reconstruction of random point patterns. *Comput. Stat. Data Anal.*, 51(2):859–871, 2006.

Zhan Shi: Points of infinite multiplicity of planar Brownian motion

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Based on the joint work with Elie Aidekon (Paris VI) and Yueyun Hu (Paris XIII)
Planar Brownian motion is known (Dvoretzky, Erdos and Kakutani 1958, Le Gall 1987) to possess points of infinite multiplicity, i.e., points where Brownian motion hits infinitely often. I am going to explain how the change-of-measures technique for spatial branching processes can help construct a family of (random) measures that are supported by the set of points of infinite multiplicity. These measures extend the ones constructed by Bass, Burdzy and Khoshnevisan (1994), and are supported by the set of thick points defined by Dembo, Peres, Rosen and Zeitouni (2001).

Ekaterina Simarova: Limit theorems for U-max statistics

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Let $\xi_1, \ldots, \xi_n, \ldots$ be i.i.d. random elements taking values in some measurable space. U-max statistics of degree $m \geq 1$ are defined by the formula

$$H_n = \max_J h(\xi_{i_1}, \ldots, \xi_{i_m}),$$

where $n \geq m$, $h(x_1, \ldots, x_m)$ is a real-valued symmetric Borel function, and

$$J = \{(i_1, \ldots, i_m) : 1 \leq i_1 < \ldots < i_m \leq n\}$$

is a set of increasing permutations of indices $i_1, \ldots, i_m$. Lao and Mayer were the first to start studying U-max statistics in their joint work [1].
We discuss limit theorems for U-max statistics in larger generality than before, namely for some broad classes of symmetric and sufficiently smooth kernels h. We also consider the applications of these theorems to the limiting behavior of stochastic geometric objects.

References

[1] W. Lao, M. Mayer. *U-max-statistics*. J. Multivariate Anal. 99(2008), 2039–2052.

Evgeny Spodarev: Spectral classification of fullerenes

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Based on the joint work with Artur Bille (Ulm University and Skoltech, Moscow) and Victor Buchstaber (Steklov Institute of Mathematics and Skoltech, Moscow)
Mathematical fullerenes C_n are simple convex 3–polytopes in a three-dimensional Euclidean space with n vertices and all facets being regular pentagons or hexagons. They serve as models for chemical carbon molecules which received much attention since the Nobel prize in chemistry awarded to Curl, Kroto and Smalley (1996) for the synthesis of Buckminster fullerene. As n increases, the number of combinatorially non–equivalent fullerenes with n vertices (called *i*somers) is growing as $O(n^9)$. This makes the problem of classification and a search for chemically and physically interesting fullerene isomers a very hard task.
In this talk, we propose a simple method for such classification based on the spectral properties of dual graphs of hexagons of fullerenes. In particular, we compute the sums of powers of eigenvalues of adjacency matrices of hexagons of a fullerene which make a hierarchical clustering possible. This clustering is highly correlated with the energy of chemical fullerenes enabling their natural ordering without complex quantum–physical calculations (such as e.g. density functional theory). The method is very fast classifying an isomer of C_n with $O(n^3 \log n)$ operations. Other related properties of fullerenes such as cospectrality, realizability of different patches of 12 pentagons, etc. are discussed as well.

References

[1] A. Bille , V. Buchstaber and E. Spodarev. *Spectral clustering of combinatorial fullerene isomers based on their facet graph structure*. Preprint, Ulm University, 2019.

[2] A. Bille , V. Buchstaber, D. Dresvyansky, T. Frauendorfer, F. Krötz, E. Spodarev and M. Willmann. *Spectral properties of combinatorial fullerene isomers*. Preprint, Ulm University, 2019.

Anatoly Vershik: Random polytopes, limit shapes, etc.

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I will describe old paper (1992) with P. Sporyshev about asymptotic behavior of f-vector of convex polytopes when dimension, and number of vertices tend proportionally to infinity. The main idea consists in so called Grassmann approach which reduced the calculations to the saddle point method for some Gaussian integrals. Then I want to formulate some new problems related to random polytopes and its applications.

Vladislav Vysotsky: How long is the convex minorant of a one-dimensional random walk?

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Based on the joint work with Gerold Alsmeyer, Zakhar Kabluchko, and Alexander Marynych
We prove distributional limit theorems for the length of the largest convex minorant of a one-dimensional random walk with independent identically distributed increments. Depending on the distribution of the increments there are several regimes with different limit distributions for the length. The most prominent of our results is a central limit theorem for the length in the case when the increments have zero mean and finite variance. Among other tools, we employ a representation of convex minorants of random walks in terms of uniform random permutations.

Elisabeth Werner: Constrained convex bodies with extremal affine surface areas

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Based on the joint work with O. Giladi (), Han Huang (Ann Arbor), C. Schütt (Kiel)
Given a convex body K in $\mathbb{R}^n$ and $p \in \mathbb{R}$, we study the extremal inner and outer affine surface areas

$$IS_p(K) = \sup_{K' \subseteq K} \big(as_p(K')\big) \text{ and } os_p(K) = \inf_{K' \supseteq K} \big(as_p(K')\big),$$

where $as_p(K')$ denotes the L_p-affine surface area of K', and the supremum is taken over all convex subsets of K and the infimum over all convex compact subsets containing K.

The convex body that realizes $IS_1(K)$ in dimension 2 was determined in [1] where it was also shown that this body is the limit shape of lattice polytopes in K. In higher dimensions no results are known about the extremal bodies.

We use a thin shell estimate of [2] to give asymptotic estimates on the size of $IS_p(K)$. We use the Löwner ellipsoid of K to give asymptotic estimates on the size of $os_p(K)$. Surprisingly, it turns out that both quantities are proportional to a power of volume.

References

[1] I. Bárány, *Affine perimeter and limit shape*, J. Reine Angew. Math. **484** (1988) 71–84.

[2] O. Guédon and E. Milman, *Interpolating thin-shell and sharp large-deviation estimates for isotropic log-concave measures*, Geom. Funct. Anal. **21** (2011) 1043–1068.

Joseph Yukich: Multivariate normal approximation for statistics in geometric probability

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Based on joint work with Matthias Schulte (University of Bern)

Given a vector $F = (F_1, ..., F_m)$ of Poisson functionals $F_1, ..., F_m$, we establish quantitative bounds for the proximity between F and an m-dimensional centered Gaussian random vector N_Σ with covariance matrix $\Sigma \in \mathbb{R}^{m\times m}$. We derive results for the d_2- and d_3-distances based on smooth test functions as well as for the d_{convex}-distance. The bounds are multivariate counterparts of the second order Poincaré inequalities of Last, Peccati, and Schulte (2016) and, as such, are expressed in terms of integrated moments of first and second order difference operators. We show that the bounds remarkably simplify when the Poisson functionals consist of sums of stabilizing score functions. We use the general results to deduce presumably optimal rates of multivariate normal convergence for statistics arising in random graphs and stochastic geometry.

References

[1] G. Last, G. Peccati and M. Schulte Normal approximations on the Poisson space: Mehler's formula, second order Poincaré inequalities and stabilization. *Prob. Theory and Related Fields*, 165: 667–723, 2016.

Sergei Zuyev: Applications of the perturbation formula for Poisson processes to elementary and geometric probability

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Based on the joint work with Günter Last (Karlsruhe Institute of Technology, Institut für Stochastik, Karlsruhe, Germany)

The binomial, the negative binomial, the Poisson, the compound Poisson and the Erlang distribution do all admit integral representations with respect to its (continuous) parameter. We use the Margulis-Russo type formulas for Bernoulli and Poisson processes to derive these representations in a unified way and to provide a probabilistic interpretation for the derivatives. By similar variational methods, we obtain apparently new integro-differential identities which the density of a strictly α-stable multivariate density satisfies.

Then, we extend Crofton's derivative formula known in integral geometry to the case of a Poisson process. Finally we use this extension to give a new probabilistic proof of a version of this formula for binomial point processes.

References

[1] G. Last and S. Zuyev *Applications of the perturbation formula for Poisson processes to elementary and geometric probability*. Available at https://arxiv.org/abs/1907.09552